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XXXIV. Difficulties in the Newtonian Theory of Light, confidered and removed, by the Rev. S. Horsley, LL. B. F. R. S.

Read Dec. 20, R. Franklin, in a letter to a correfpondent at New-York, the 23d of that valuable collection with which the public was obliged in the latter end of the year 1768, proposes some objections to the Newtonian theory of light. His words are these. " I am much in the dark about " light. I am not fatisfied with the doctrine that " supposes particles of matter called light, continual-5' ly driven off from the fun's furface with a swiftness " so prodigious. Must not the smallest particle con-" ceivable have, with fuch a motion, a force exceed-" ing that of a twenty-four pounder discharged from " a cannon? Must not the sun diminish exceedingly by fuch a waste of matter, and the planets recede to greater distances by the lessened attraction? Yet " these particles with this amazing motion will not " drive before them, or remove, the least and lightest " dust they meet with: and the sun, for aught we " know, continues of his antient dimensions, and his attendants move in their antient orbits." Dry Vol. LX. Hhh Franklin's Franklin's questions are of some importance, and deserve a strict discussion.

Upon the supposition that light is a copious emanation of innumerable small particles of matter from the sun, I had once occasion to enquire, what the force of motion produced in every such emission could possibly amount to at the utmost.

For this purpose, I made an estimate of the greatest probable magnitude of the particles of light; and of

the greatest density of each.

I likewise computed the greatest number of such particles, that could possibly fly off at once from the surface of the sun; supposing the sun's horizontal parallax to be no more than 8".

These computations, with an account of the principles on which they were founded, having been already given to the public^a; I shall make use of the results (which I shall here briefly state) as data for the discussion of Dr. Franklin's questions.

I suppose the particles of light to be equal spherules. This, perhaps, is not the case. Each color has probably its own size; but, there will be a mean size,

which is fufficient for my purpose.

This mean fize I suppose to be so small, that the diameter of each spherule does not exceed one millionth of one millionth of an inch. I shall shew hereafter, that there is much reason to suppose that the particles of light are in fact much less than spherules of this diameter.

In a little treatise, entitled, The Power of God, deduced from the computable instantaneous Productions of it in the Solar System.

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I suppose the density of each particle three times that of iron^b. The number of such spherules that contain as much matter as an iron ball of one yard diameter, I have found to be 15552 XXXVI^{cd}; conquently 576 XXXVI such spherules contain as much matter as an iron ball of I foot diameter.

Let fuch a ball be supposed to move uniformly with a velocity that should carry it 1000 yards in 1".

The light of the fun traverses the semidiameter of the orbis magnus in 7' nearly.

In the ensuing calculations, I shall reckon the fun's

horizontal parallax 9".

According to this hypothesis, the semidiameter of the orbis magnus will contain 22919 semidiameters of the earth.

In 1" of time, light, according to the velocity affigned to it, traverses $\frac{1}{+20}$ th of this space, or 54,57 semidiameters of the earth, or 381092323 London yards.

Hence the velocity of each particle of light, will be to that of the iron ball moving 1000 yards in

1" as 381092 to 1, very nearly.

And the ball being I foot diameter, it has been shewn that the matter in each particle is to the matter in the ball, as I to 576 xxxvI.

Hence the force of the motion in each particle of light, is to the force of motion in the ball as 381092 to 576 xxxvI; that is, as I to 1511444 xxvII, or,

· Ibid.

b Instantaneous Product. &c. p. 30.

The Roman numerals placed after the Arabic characters, denote the number of zeros that must be added to the Arabic figures, to compleat the expression of the number intended.

it is equal to the force of motion in an iron ball of 1 foot diameter moving $\frac{1}{1511444 \times x \times v \times 1}$ th of 1000 yards in 1", or to that of an iron ball of 1 inch diameter moving $\frac{1}{874679 \times x \times 1}$ th of 1 yard in 1", or to that of an iron ball of $\frac{1}{4}$ of an inch diameter moving $\frac{1}{13666 \times x \times 1}$ th of 1 yard in 1"; that is, moving less than $\frac{1}{4555 \times x \times 1}$ of 1 foot in 1"; that is, moving less than a foot in 4555 xx.1 seconds, or in more than one bundred forty four thousand millions of millions of Egyptian years: or the force of motion in each particle of light coming from the sum of the function in each particle of less than an inch in 12 thousand millions of millions of Egyptian years.

Dr. Franklin's first question is answered. A parficle of matter, which is probably larger than any particle of light, moving with the velocity of light, has a force of motion, which, instead of exceeding the force of a twenty-four pounder discharged from a cannon, is infinitely less than that of the smallest shot discharged from a pocket pistol, or less than any that art can create.

I proceed to the other questions—And, I think that I shall make it appear, that it is very possible that light may be produced by a continual emission of matter from the sun, without any such waste of his substance as should sensibly contract his dimensions, or sensibly alter the motions of the planets, within any moderate length of time.

Indeed, I do not think it necessary to the production of any of the phænomena of light, that the emanation emanation from the sun should be continual in a strict mathematical sense, or without any interval. It seems sufficient to all purposes, that the intervals should be exceeding short. But this I only mention.—I think it possible that a continual emanation might subsist, without any such dangerous consequences to the solar system, as Dr. Franklin apprehends. Dr. Franklin's character is not more distinguished by his superior talents, than by a candor truly philosophical. And upon this circumstance, I build the strongest considence, that he will not be offended, that I differ from him: that, as a friend to enquiry, he will be pleased that I take the liberty to communicate my own notions, however opposite they may be to his.

It will be easily understood, that a continual emanation from the fun does not necessarily imply a continual waste, or loss, equal to the emanation.—If light is continually iffuing from the fun in all directions, part of this is continually returning to him, by reflection from the planets, and other light is continually coming to him, from the funs of other fystems. It is true, that the light which he receives. is but a very small part of the light which he gives. For if the light always coming to the fun were equal to the light always going from him, our atmosphere would be as strongly enlightened in the night as in the day.—But this is not the case; and the proportion of the light that comes, to the light that goes, cannot be greater than that of night-light at a medium, to day-light at a medium—still it is something—and the continual loss of substance that the sun sustains cannot be more than the difference between the light that he gives, and the light that he receives.

And

And therefore, if there were no other recruit of the fun's substance (which is by no means a probable supposition) yet the continual waste will, on this account alone, be less than the continual emission; and the sun cannot lose so much of his substance as a single emission of light contains, but in some determinate time. I shall suppose that the sun gives so much more than he receives, that he loses the amount of one emission in every second of an hour. Let us see what will be the consequence. Every particle of light that issues from the sun, must leave a spherical vacuity of one millionth of one millionth of an inch diameter.

The greatest number of particles of this size that can issue from the surface of the sun at once, if the horizontal parallax be 9", is 104666 x11°, because this is the greatest number of such particles that would have room to lie at once upon his surface. And the same will be the greatest number of spherical vacuities made by one emission.

Many of these vacuities are no sooner made, than they are filled up by the light that is coming to the sun from other systems, or returning to him from the bodies of his own, or, perhaps with other matter which he may receive in various emanations from the planets; for I strongly suspect that a perpetual circulation of siner matter may subsist between all the large bodies of the universe. The emission of light, however, is supposed so far to exceed the whole

The greatest number of particles that can issue from the fun at once, was reckoned in the Instant. Product. 132467 xLI; but then the ⊙'s parallax was reckoned only 8".

fupply, that in every fecond, over and above the vacuities that have been filled up with adventitious matter, the foregoing number, 104666 xLI, have been formed that have received no fuch supply. The fluid matter of the sun frushes from all sides into these, and fills them up to its own level. The sun by this means shrinks a little, and loses, once in every second, so much of its solid content, as the solid contents of these vacuities amount to.

I have found that the folid contents of 46656 xxxv1 fuch vacuities are equal to the sphere of one yard 8.

Therefore the folid contents of 104666 xLI such vacuities, are equal to 224335½ times the sphere of one yard. And so much is the sun's solid content diminished every second.

The sphere of the earth is to the sphere of one yard, as 27247031 XIV to 1 h.

Therefore the sphere of the earth is to 224335½ times the sphere of one yard as 121456 x1 to 1.

Therefore the fun loses an earth of its solid content in 121456 x1 seconds, or 385130000 Egyptian years nearly.

If the sun's parallax be 9'', the solid content of the earth is $\frac{1}{12.171+20}$ of the solid content of the sun.

Therefore, in 385130000 Egyptian years, the sun should lose $\frac{1}{1247420}$ th of his solid content, and confequently in the same time the diameter of the sun

f I suppose the sun to be a shuid mass: by this hypothesis, I give the utmost force to Dr. Franklin's objection; sor, the more persectly the sun is shuid, the more suddenly will the vacuities be filled up.

⁸ Vide Instant. Product. p. 30.

h Ibid. p. 16.

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should lose 1 th of its whole dimensions. And in the same time the apparent diameter should lose the like part of its quantity, if the distance between the earth and sun remained unaltered.

The sun's mean apparent diameter contains 1922 seconds. Therefore $\frac{1}{3032200}$ th of the 0's apparent diameter, is $\frac{1}{1900}$ th of one second very nearly. So inconsiderable would be the whole diminution of the sun's apparent diameter, that could arise from the waste of his substance, in 385130000 Egyptian years.

But the waste of the sun's substance must lessen the attraction between the earth and sun. As the attraction lessens, the earth will recede to greater distances. And hence there will arise a further diminution of the sun's apparent diameter, and a pro-

longation of the anomalistic year.

The density of each particle of light has been supposed three times that of iron, or 23 times the mean density of the earth. Therefore, as often as the sun's loss by light amounts to an earth in size, it will amount to 23 earths in matter.

The matter of 23 earths is $\frac{1}{\sqrt{3}}$ d of the fun's matter, if the fun's parallax be 9".

Therefore in 385130000 Egyptian years the sun loses $\frac{1}{13232}$ d of his matter; and the gravitation towards the sun, at any given distance, diminishes in

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I reckon the mean density of the earth no greater than that of common water. It is certain that it cannot be less. Sir Isaac Newton reckons it 5 or 6 times greater; but I confess that I am not satisfied with his reasons for making it so great. If I have under-rated it, I have, in so doing, given the advantage to Dr. Franklin's objection.

the fame proportion. But this alteration is much too small to discover itself in the motions of the earth or any of the planets. I will not at present consider, by what law the distance of the planets from the fun would increase, because the enquiry could not be reduced to a small compass; but it is obvious, that, whatever that law may be, it must arise solely from the diminution of gravitation: and the like is to be faid of the increase of the anomalistic periods. therefore, while the diminution of gravitation is infensible, the changes in these circumstances must be infensible too. Of all the changes to which our system may be obnoxious, those which should arise from the waste of the sun's substance in light, upon the supposition that light is an actual emanation of matter from the fun, reckoning that waste at the utmost, are perhaps the least considerable.

In the foregoing computations, the instantaneous emission of light has been greatly over-rated. For if the particles of light were of the magnitude and density which has been assigned to each, and were to issue from the sun in the close arrangement that has been supposed, they would form a sort of crust upon the sun's surface, at least 12 times more dense than water, i. e. 9600 times more dense than our atmosphere in the parts next the earth's surface, if the density of common water compared to that of air be reckoned only as 800 to 1 k. But if the density of light upon the sun's surface be 9600 that of our air, its density when it arrives at the earth, or its

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^{*} It is well known that the denfity of water to that of air, is as 850 to x at leaft.

denfity upon the furface of the orbis magnus 1/2 should be more than the part of the density of our air. When substances of different specific gravities, as a piece of gold and a piece of cork, descend, in the exhausted receiver, with equal velocities, and fall equal heights in the fame time, it is obvious, that the density of the medium, through which they fall, bears no sensible proportion to the density even of the lighter substance. The medium through which such substances fall, in a transparent glass receiver, is composed of fome small portion of rarified air, and, to all appearance, of the same quantity of light, as the receiver contains before exhaustion. For the quantity of light, in a transparent receiver, can by no means fuffer any diminution, by the action of the air-pump. The denfity of the light therefore, in our air, is certainly too fmall to bear any fensible proportion to that of gold or even of cork. And the denfity of cork bears, though a great, yet a finite and a sensible proportion to that of the atmosphere; because in air gold and cork do not descend with equal velocities. Hence, I think, I may conclude with the greatest certainty, that the density of the sun's light at the earth, or upon the surface of the orbis magnus, is too small to bear any fensible proportion to the density of common air; and I shall hardly underrate the denfity of the light, if I reckon it only part of the denfity of the air, or socooth part of the denfity of common water. Suppose this

¹ By the furface of the orbis magnus, I mean to denote a particular place in absolute space, namely the surface of that sphere which is concentric with the \odot , and hath the earth's mean distance from the \odot for its semidiameter.

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to be the density of light upon the surface of the orbis magnus, and it will be found by computation, that its density upon the surface, must be less than $\frac{z}{1.7.3}$ d of the density of common water.

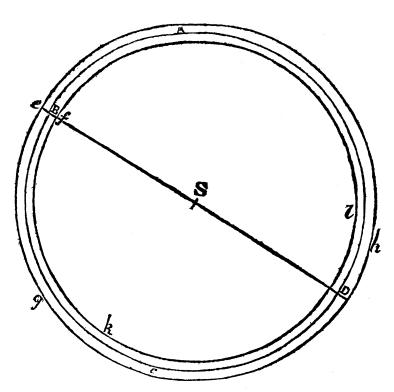
From these considerations, I think it may be concluded with the greatest certainty, that the quantity of matter that issues from the sun in light, has been greatly over-rated in the foregoing computations.

I apprehend, however, that the density of each feparate particle cannot be less than has been supposed: but that the magnitude of each is less, and the arrangement less compact. Let the density of each, and the number that issues at any one time from the fun, remain as before; and let us confider, in what proportion the magnitude of each particle, must be diminished, so that they may altogether form a fluid, on the sun's surface, 173 times less dense than water. Let ABCD represent a section of the sun's fphere, through the centre Sn. In the fpherical furface ABCD, take any point B. Join SB, and take Be, Bf, each equal to the semidiameter of a particle of light. Upon the centre S, at the distances Se, S f, imagine two other spheres, e g b, f k l, one inclosing, the other inclosed within the sphere ABCD. The folid space e f k l g b, is the space that contains all the particles of light (with their interstices) that issue together from the spherical surface ABCD; and because ef bears an exceeding small proportion to SB, therefore the spherical surfaces e g b, f k l, are very nearly equal each to the other, and to the sphe-

M Vide Instant. Product. p. 37. See the fig. p. 429.

rical surface ABCD; and the solid space contained between them is very nearly equal to a cylinder, upon a base equal to the spherical surface ABCD, and of altitude equal to ef. Therefore diminish the diameter of each particle of light, that is, diminish ef, in any proportion whatsoever, and the solid space ABCD xef diminishes in the same proportion. And if the matter in that space were given, the density of it would always be as $\frac{I}{ef}$. But the matter of each particle of light, and consequently the matter of the given number in the space ABCD xef, is always as ef^3 . Therefore the density of the substance formed by light in the space ABCD xef, is always as ef^3 , or as ef^2 .

That is, the number of spherical particles of light, in the folid space ABCD xef, being given, and the density of each particle, and the spherical surface ABCD being given, the square of the diameter of each particle will be as the denfity of the substance they compose, upon the surface of the sun. But it has been found, that if the diameter of each spherule were one millionth of one millionth of an inch, the greatest number of such spherules, that the sun's furface can contain at once, would, with the denfity which has been affigned to each separate particle, form a substance upon it 12 times more dense than Therefore, that the same number of such water. particles should form a substance about 173 times less dense than water, the diameter of each must exceed 10 th of one millionth millionth of an inch. But I have shewn that the greatest



greatest probable density of light upon the sun's surface does not amount to 173d part of the density of common water. Therefore the diameter of each spherule does not exceed \(\frac{1}{4356}\text{th}\), or more accurately, \(\frac{1000}{435643}\text{th}\), of one millionth of one millionth of an inch; and is probably still less \(\frac{1}{100}\text{th}\). More than 95100 spherules of this size go to make up one of the size sirst assumed. Therefore, though the sun should lose as much

Instant. Product. p. 38-41.

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matter, as can be supposed to be contained in any single emission of light, 95100 times in every second, no sensible alterations in the system could take place in millions of years. And perhaps light does not issue from the sun so frequently as 95100 times in a second.

June 13, 1770.

S. Horsley.

P. S. The late Dr. Pemberton, of Gresham College, to whom the foregoing paper was communicated, about twelve months before his death, in a letter with which he favoured the authour, upon the subject, remarked, that he had no material objection to any part of the reasoning, except it was, that the particles of light are too peremptorily spoken of as spherules. Their real figure is quite unknown; but the probability is, that they are not spherical, since Sir Isaac Newton found that their different sides have different properties. As the like objection may occur to others, it may not be improper to add a few words to obviate it. I would by no means be understood to affert, that the particles of light are of a spherical figure. But, whatever their figure may be, I conceive that their fixe must be so small, that the diagonal of each little folid cannot exceed one millionth of one millionth of an inch, or, at least, that

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that each is capable of being circumscribed within a sphere of that diameter. To the reasons which are given for making them so small, in the treatise so often referred to, I shall here add another, which may perhaps be more generally convincing: namely, that these bodies must be so minute as to find room to enter in, in swarms, at the pupils of the eyes of the smallest microscopic animals, and not to injure, by their stroke, the very delicate fibres of their optic nerves, nor to lacerate the edges of the uvea; which those that enter near the fides of the perforation, if their figure be not round, must often brush with their angles, as they pass by. Now, if it be granted, that the greatest diagonal of each solid corpuscle of light does not exceed one millionth of one millionth of an inch, or that each is capable of being circumscribed with a sphere of that diameter; then, the solid content of a sphere of that diameter is the maximum of the folid content of each corpuscle, and the matter in such a sphere, of due density, is the maximum of the matter in each corpuscle. The maximum therefore of the force of motion in each particle, is the force of a spherule of the fize assumed, moving with the known velocity of light; and therefore, be the figure what it will, my conclusion (p.420.) which rests entirely on the maximum of fize and matter, will still hold good, unless it can be shewn that I have under-rated the density of each particle; and even if it could be proved that I have assumed the density too fmall, in the proportion of 1 to 12 thousand times the square of one million, still the general conclusion will not be shaken: for this vast increase of the denfity will raise the force of motion, in each corpuscle

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puscle, to no more than that of an iron ball \$\frac{1}{4}\$ of an inch diameter, moving one inch in a whole

year.

Again, in the diagram, p. 429, SB being the diameter of the sun, and f B, Be being each the half of one millionth of one millionth of an inch, the space contained between the spherical surfaces fkl, egh, is the maximum of the space that the particles of light with their due proportion of interflice can fill, as they fart forth from the surface of the sun. For, whatever their figure be, it must be such a one as can be laid between these two spherical surfaces. Now the quantity of matter in this space, must not be greater, than it would be upon the hypothesis that each figure was spherical, and the number of spherical particles the greatest possible. Since, upon that hypothesis, the density of matter, crowded into this space, is vastly too great, to be consistent with the appearances of nature (vide p. 425, 426.); and confequently a greater density would be more inconfiftent. Therefore the maximum of the matter, and confequently, if the denfity of each feparate particle has been rightly assumed, the maximum of the solid content, in each emission of light, is what I have made it; at least it does not exceed what I have made it. be the figure of the particles what it will; and my conclusions (p. 424 and 425.) which rest entirely upon the maximum of the folid content, and that of the matter, and are the stronger the less these maxima be, will still hold good. But if these conclusions stand, the objections moved by Dr. Franklin vanish.

The only part of my reasoning which will be affected, by supposing the figure of the particles of light not to be spherical, will be that, (p. 427, &c.) in which I attempt to shew, in what proportion the magnitude of each particle, and the matter, contained in each emission, must be less than the maximum, in order to make the denfity of light no greater, than may be confistent with the appearances of nature. Here indeed the figure of the corpufcles is of great importance. The diminution necessary will be very different in different figures, and the figure, I confess, may be fuch, as to make it much less, than what I have shewn to be requisite upon the spherical hypothesis. However, if -1 to part of the density of our air be admitted to be as great a denfity, as can reasonably be allowed to light, at the furface of the orbis magnus, the matter of each emission must not, upon any hypothesis of the figure of the corpuscles, exceed th of the maximum.

For, in order to bring the density down, from the maximum to any other given limit, either the matter must remain unaltered, and the space, which it is supposed to occupy, be encreased, in the proportion in which the density is to be diminished; or the space must remain unaltered, and the matter be diminished in proportion to the density; or, if both space and matter be altered, the matter must be changed in the proportion compounded of the two, in which both space and density are varied. Now the space which we suppose the light to occupy, as it is emerging from the surface of the sun, must not exceed the space contained between the spherical surfaces egb, fkl (sig. p. 429.). For that space, as has been Vol. LX.

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observed, is the maximum. This space may be diminished; but if it be diminished, the matter being diminished as the space and as the density jointly, must be more diminished than in the simple proportion of the denfity. Therefore the diminution of the matter will be the least possible, if, the space being supposed to continue at its maximum, the matter be diminished in the simple proportion of the maximum of the denfity to the denfity required. Various formations of the particles of light might be thought of which would answer this purpose. It might be answered, indeed, upon the spherical hypothesis, by diminishing the number of spherules, retaining the maximum of their fize; or retaining the maximum both of fize and number, if it can be thought reasonable to diminish the density of each particle. But upon any hypothesis, the diminution of matter cannot be less than has been said. That is, the matter of each emission cannot possibly exceed 1/20 8 th of the maximum. For the proportion of the maximum of the denfity to the denfity required, will be found by computation, to be that of 2084 to 1, very nearly. Therefore the utmost probable amount of one is 1 th of the maximum, be the figure of the corpuicles what it will. And the fun may lose the quantity of a whole emission 2084 times in a second without any fensible consequences.

I cannot apprehend from, any quarter, so unphilosophical an objection, as that the extreme minuteness of the particles of light, which I have shewn to be necessary, if light be really matter, is an argument against their existence. Size is a meer accident, and no part of the essence of any being. Great and small, applied to finite things, are purely terms of comparison. One Being only is absolutely Great: he whose substance pervades and fills the whole and every part of Absolute Space; because in respect of Him, all things that are, are little.

Notwithstanding the maximum of moving force, in each particle of light is so inconsiderable as I have shewn it to be, yet the number of particles, out of each emission, which are directed towards the earth, and fall upon its enlightened hemisphere, being exceeding great, it may perhaps be imagined, that the, force impressed by them all upon the earth, if they all actually strike its surface, may amount to something worth attending to. I have taken the pains to satisfy myself upon this question, and shall briefly mention the result of my computations.

Reckoning every emission at its maximum, I find that the number of the particles, out of each, which should fall upon the earth's enlightened hemisphere, is 492023xxx1, or that of which the logarithm is 36.6919854. The moving forces of this whole number amount to as much, as the force of an iron ball, of one yard diameter, slying 68 mile and 887 yards in one second. But the progressive force, which they might communicate to the earth, does not exceed that of an iron ball of the same size, slying 34 mile and 443 yards in one second. And, if this whole force be transferred

Only one particle out of each emission from any very small given part of the sun's surface, can strike the earth's surface per-K k k 2

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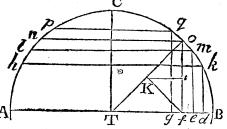
from the iron ball to the globe of the earth, the progressive velocity of the earth's center will be found to be no more, than that with which a body would traverse about $\frac{1}{2}$ of one millionth of one millionth of

pendicularly. The force of every particle, which impinges obliquely, is refolvable into two, one perpendicular to the furface, and the other parallel to it. That part of the force, which is perpendicular to the earth's furface, produces a progreffive motion of the whole globe. The other, which is parallel to the furface, contributes nothing to the progressive motion, but tends to produce a rotation of the globe upon an axis. Hence the progressive force of motion, communicated to the globe, is less than it would be, if all the particles struck the furface perpendicularly. It is further leffened upon another account, namely, that the whole of th sperpendicular force is not effective in moving the earths center. I find by computation that the diminution upon the whole is $\frac{1}{2}$. For the ease of the mathematical reader, I shall briefly state the principles upon which this computation of the force with which the earth may be struck by light, has been framed.

The number of particles, which are directed to the earth out of each emission, is to half the number of the whole emission, in the duplicate proportion of the chord of the sun's horizontal parallax, to the chord of 90°: by the known doctrine of Archimedes. Hence, the number of the whole emission being determined, the number of those which tend towards the earth, is given. And the force of each single particle being given, the sum of the forces of that given number is given; and this is the progressive force which would be impressed upon the earth, if all the force of each particle were effective. In what

proportion the progreffive force is diminished, on account of the various obliquities of impulse, is thus investigated.

Imagine T to be the earth's center and ACB to be half a great circle A



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an inch in 100". This is the utmost effect of the force impressed upon the earth by each emission. If the emissions were incessant, this might be considered as a central force, counteracting the sun's attraction:

of the earth, perpendicular to that which separates the enlightened and the dark hemisphere, and which shall be called the termi-Upon the plane of this semicircle, suppose the terminator, and its parallels in the enlightened hemisphere, to be projected, into right lines AB, kb, lm, no, &c. which are diameters of the circles respectively, of which they are the projections. The fun's distance may be considered as infinite; and therefore the rays of light, i. e. the directions of the particles, when they reach the earth's furface, are to be confidered as parallel to each other, and all of them perpendicular to the plane of the terminator. Now imagine the whole enlightened hemisphere to be divided into innumerable little zones AB k h, h k m l, m l n o. &c. by small circles parallel to the terminator. Let the breadths of these little zones, measured upon a great circle passing through the poles of the terminator, that is, let the infinitefimal arcs Bk, km, no, &c. be so proportioned to each other, that perpendiculars kd, me, of, &c. being drawn from the extremities of these arcs, to the right line AB, which is the common interfection of the great circle ACB, and the plane of the terminator, the infinitesimal segments of that line Bd, de, ef, &c. may be equal. Now imagine the particles of light which fall upon any one of these little zones, for instance, nogp, to meet with no refistance from the earth's surface, but to penetrate the globe, and to pass on without refraction or inflection, in the direction perpendicular to the terminator, till they arrive at the plane of the terminator, and there suppose them to stop, and each to lye still, in the place on which it falls. It is evident that the particles of light that fall upon, and have been supposed to pass through, the spherical zone p q on, will, with their proper interstices, cover that annular space upon the plane of the terminator, which is the orthographical projection of the zone p q o n, upon that plane, and is comprised between the circumferences of circles, of which the right lines Tg and Tf are the radii. Hence the number of the particles of light, which fall upon the evanescent zone pq on, are as that evanescent annular space

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for its tendency is to push the earth directly from the sun. I need not say, that it is infinitely too small, in comparison of the sun's attraction, to produce any sensible effect.

which they cover, that is, as $g f \times$ the circumference of the circle of which $\hat{\mathbf{T}}$ f is the radius, that is, as $gf \times$ in the right line $\mathbf{T} f$. But that part of the force of each particle, impinging on the zone $p \neq q \circ n$, which is perpendicular to the furface of the zone, is as of, if TB (the semidiameter of the earth) be put for the whole force. For join To, and draw f K perpendicular to To. The particle impinging at o moves in the direction of. Let the right line of then express its whole force, and this force of is composed of the two oK, Kf, of which oK is perpendicular to the surface of the sphere at o, and K f is parallel to it. oK: of = of: oT or TB. Again, through K, draw Kt perpendicular to of. The force oK is resolved into two ot, tk, of which ot is perpendicular to the plane of the terminator, and is the only part of the force oK, which tends to produce a progressive motion of the globe, in the direction of the impinging particles, that is, directly from the fun. The other part t K urges the center of the globe along the plane of the terminator; but the forces t K being equal and contrary on opposite fides of, and at equal distances from, the perpendicular ray, de-Aroy each others effects. Now it has been shewn, that the whole force of the particle impinging at o is to that part of its force which is perpendicular to the earth's surface at o, as TB to of. And it is manifest that oK is to ot, that is, the perpendicular force, of the particle impinging at o, is to that part of it, which is effective in moving the earth's center, as TB to of. Therefore the whole force of the particle impinging at o is to its effective part, as TB² to of^2 . That is, the effective part is as of^2 . The number of particles impinging on the zone, has been shewn to be as $gf \times Tf$. The progressive force of motion excited in the earth's center, by all the particles impinging on the infinitessimal zone pqon, must be, as the number of the particles and the effective part of each, jointly; that is, as $gf \times Tf \times of^2$, or writing a for TB, and x for Bf, as $\dot{x} \times a - x \times 2ax - x^2$. And this is the fluxion of the progressive force of motion excited in the globe, by the particles impinging upon that fegment of the

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The rotatory forces mentioned in the last note if they were infinitely greater than they really are, would not, in the least degree, disturb the diurnal rotation; because every one of them is destroyed, by an equal one, in a contrary direction, on the other side of, and at an equal distance from, the perpendicular ray.

I have enquired, what may be the utmost stroke, which the retina of a common eye suftains, when the eye, in a bright day, is turned up directly to the fun. This force will evidently be at its maximum, if the emission be reckoned at its maximum. The number of particles which enter an eye, looking up directly at the fun, are to the number out of each emission, which are directed towards the earth, in the duplicate proportion of the diameter of the pupil to the diameter of the earth. And the force with which the eye is struck, is to the fum of the forces of all the particles which strike the earth, in the same proportion. If, therefore, the diameter of the pupil, when the eye is exposed to the direct impulse of the sun's rays, be reckoned th of an inch, which I apprehend must rather exceed

fphere, of which p ABq is the projection. The number of particles impinging on the zone p q o n, being as $gf \times Tf$, or as $\dot{x} \times \overline{a-x}$, if each impinged perpendicularly, and its whole force were effective, the fum of the effective forces impressed upon the whole, would be as $gf \times Tf \times TB^2$, or $\dot{x} \times a^3 - a^2x$. And this would be the fluxion of the progressive force of motion of the globe excited by the particles impinging upon the segment of which p ABq is the projection, if all impinged in directions perpendicular to the surface, and the whole of their forces were effective. The fluent of $\dot{x} \times a - x \times 2ax - x^2$ is $\frac{1}{4} \times 2ax - x^2$. And the fluent of $\dot{x} \times a^3 - a^2x$ is $a^3x - \frac{1}{2}a^2x^2$. than

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than fall short of its real magnitude, in those circumstances, it will be found that every stroke which it receives from them, exceeds not that, which an iron shot, \(\frac{1}{4}\) of an inch diameter, would give, moving only at the rate of 16,16 inches in a year. This would be the stroke if the emission were at its maximum. Is it not owing to the extreme minuteness of the fibres of the nerves, that a stroke, which is certainly less than the \(\frac{1}{2\cdot 2\cdot 4}\) th part of this, is not sustained by our organs, without pain?

When x=a, the first of these two fluents is the sum of the progressive forces actually impressed upon the whole hemisphere ACB, and the latter is the sum of the forces which would be so impressed, if all the impinging particles impinged perpendicularly, and the whole force of each were effective. But when x=a the first fluent becomes $\frac{1}{4}a^{4}$. And the latter becomes $\frac{1}{2}a^{4}$. Whence it is manifest that the progressive motion communicated to the globe of the earth, by the particles of light, is to the force which they would communicate, if the whole force of each were effective, in the proportion before assigned, of one to two.